A Note on Operating Leverage and Expected Rates of Return

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Abstract

Conventional wisdom is that greater operating leverage increases systematic risk and therefore leads to a higher expected rate of return earned by a firm's owners. This paper shows that the relationship between operating leverage and the expected rate of return is actually non-monotonic when allowance is made for the option to abandon an unprofitable project: the expected rate of return is an increasing function of operating leverage when the latter is low, but a decreasing function when it is high. This demonstrates the dangers in drawing inferences from models that ignore the flexibility embedded in typical investment projects.

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1 Introduction

Compared to many areas of corporate finance, relatively little is known about how the characteristics of a firm affect expected returns earned by security-holders. Early papers used the Capital Asset Pricing Model (CAPM) to calculate expected rates of return when firms' cash flows were derived from firm characteristics. For example, researchers in corporate finance found that a firm's beta is an increasing function of its operating leverage (Lev, 1974; Mandelker and Rhee, 1984) and researchers in industrial organization found that greater market power lowers systematic risk (Subrahmanyam and Thomadakis, 1980). However, this early research was constrained by the limits of the CAPM, and the analyses were typically static. Interest in the determinants of expected rates of return has revived as researchers apply new modeling techniques. For example, Berk et al. (1999), Gomes et al. (2003), Carlson et al. (2004), and Cooper (2006) show that the relative importance of assets-in-place and growth options varies over time as firms invest, leading to predictable changes in expected returns.

In this paper I use real options analysis to reassess the earlier work on the importance of operating leverage for expected rates of return. The definitions of operating leverage vary, but most corporate finance textbooks discuss its relationship with expected rates of return, and the explanation offered for a positive relationship between the two is usually along the same lines:[1] all else equal, a firm with high operating leverage has high fixed costs and low variable costs; the revenue from one additional unit of production is therefore offset by a relatively small increase in variable cost, so that the firm's cash flow is especially sensitive to underlying demand; as a result, systematic risk is high. However, this argument ignores the real option to abandon an unsuccessful project, which is surprising since for any firm with high fixed costs the prospect of future abandonment must surely be important.

I address this shortcoming by interpreting a firm with fixed costs as a portfolio comprising (i) the firm with the abandonment option removed and (ii) the abandonment option on its own. The firm's expected rate of return is therefore a value-weighted average of the expected rate of return of the first asset and the expected rate of return of the abandonment option (which is less than the risk-free interest rate). I use this decomposition to identify the two components of the change in the expected rate of return when operating leverage increases. First, as the

[1] For example, Damodaran (2001, pp. 202–203) and Ross et al. (2002, p. 316–317) define operating leverage as the elasticity of operating profit with respect to changes in sales. Brealey and Myers (1991, pp. 199–200) do not formally define operating leverage but present a simple derivation that the CAPM asset beta is an increasing function of the ratio of the present value of fixed costs to the present value of the asset, holding the revenue beta constant.

conventional wisdom suggests, the expected rate of return corresponding to the no-flexibility case rises; this increases the weighted-average of the two costs of capital. Second, the relative importance of the abandonment option will change—if the change in cost structure makes the option to abandon more valuable, the effect is to shift weight from the relatively-high expected rate of return on the first asset to the relatively low one on the abandonment option, which reduces the weighted-average. In general, increases in operating leverage therefore have an ambiguous impact on the expected rate of return of a firm. Analysis of a particular example shows that the second effect can dominate the first when operating leverage is high. That is, the relationship between operating leverage and the expected rate of return can be non-monotonic.

Lev (1974) analyzes the relationship between operating leverage and the expected rate of return by fixing the output price and the variable cost per unit of output and allowing exogenous random output. He defines operating leverage as the ratio of fixed to variable operating costs and shows that the CAPM beta is higher for firms with greater operating leverage (actually, with lower variable cost per unit of output). Mandelker and Rhee (1984) show how a firm's asset beta can be decomposed into the product of the degree of operating leverage, the degree of financial leverage, and the amount of "intrinsic business risk". They define the degree of operating leverage to be the elasticity of earnings with respect to changes in production.[2] The common feature of these papers, which is also present in the textbook treatments, is that the models are static in nature. Only when the firm is modeled in an intertemporal setting, and the embedded flexibility incorporated, is the full picture revealed. This paper therefore provides an example of how intertemporal aspects of investment can overturn familiar static results.[3]

Several studies have investigated the empirical relationship between the expected rate of return and different measures of operating leverage. For example, Lev (1974) finds that higher average variable costs are associated with lower equity betas for US steel producers; the relationship is weaker (but has the same sign) for electric utilities and oil producers. Mandelker and Rhee (1984) find a statistically significant positive relationship between the equity betas of US manufacturing firms and their elasticities of earnings with respect to changes in sales. Sudarsanam (1992) finds that the equity betas of UK firms during the mid-1970s are a decreasing function of the capital–labor ratio and an increasing function of the ratio of fixed assets to sales. However, none of these studies allow for a non-monotonic relationship between beta and operating leverage. One study that comes close is Jose and Stevens (1987), who find that,

[2] Subrahmanyam and Thomadakis (1980) endogenize the firm's choice of capital and labor and find a negative relationship between the capital–labor ratio and the firm's expected rate of return. It is tempting to associate the capital stock with fixed costs and labor with variable costs and interpret their finding as saying that there is a negative relationship between operating leverage and the expected rate of return. However, Booth (1991) shows that the degree of operating leverage (measured by the elasticity of earnings with respect to changes in the output price) is actually negatively related to the capital–labor ratio, so that the positive relationship between the expected rate of return and the degree of operating leverage is restored.

[3] Boyle and Guthrie (2003) give another example; they show that capital market frictions can actually lead to over-investment when investment timing flexibility is considered. This contrasts with the standard static result that capital market frictions can lead to under-investment (Myers and Majluf, 1984).

consistent with the model in this paper, the asset betas of a sample of US firms are increasing in the capital–labor ratio for low levels of this ratio, and decreasing in the capital–labor ratio for high values.

In the next section I describe a simple model of a firm that can abandon production, derive the firm's expected rate of return, and show how changes in the firm's cost structure affect both its operating leverage and its expected rate of return. In Section 3, I use a particular cost structure to demonstrate that increases in operating leverage can be associated with reductions in the expected rate of return. Section 4 concludes.

2 A simple valuation model when operations can be abandoned

2.1 Cash flows

I consider a firm that incurs a cost of $f + v(x)$ whenever it produces output at the rate x, where x can take any nonnegative value, f is the constant fixed cost of production, and v is an increasing strictly convex function of x satisfying $v(0) = 0$. There are no costs associated with adjusting the level of production and the owner of the firm has the option to permanently abandon it at any time; the firm's salvage value equals the constant S. There are no taxes and the firm is all-equity financed. The output price evolves according to the geometric Brownian motion

$$dp_t = \mu p_t dt + \sigma p_t d\xi_t,$$

where μ and σ are constants and ξ_t is a Wiener process. The risk-premium for bearing p-risk equals $\lambda > 0$, the risk-free interest rate equals r, and these constants satisfy $r + \lambda > \mu$.

If the firm is still operating at date t, the owner will choose x_t to maximize $p_t x_t - f - v(x_t)$. Thus, output at date t, $x(p_t)$, is defined implicitly by $v'(x(p_t)) = p_t$, and the firm generates a cash flow of

$$\pi(p_t) = p_t x(p_t) - f - v(x(p_t))$$

at date t.

The simplest measure of operating leverage is the ratio of fixed to variable costs (Lev, 1974),

$$\text{OL}_1(p) = \frac{\text{fixed cost}}{\text{variable cost}} = \frac{f}{v(x(p))}. \tag{1}$$

An alternative measure of operating leverage is the elasticity of earnings with respect to changes in some measure of production. For example, Mandelker and Rhee (1984) define operating leverage as the percentage change in earnings that is associated with a one percent increase in the number of units of output. Since output is a function of the output price, the implied measure of operating leverage is

$$\begin{aligned}
\mathrm{OL}_2(p) &= \frac{\text{output}}{\text{earnings}} \cdot \frac{d\,\text{earnings}}{d\,\text{output}} \\
&= \frac{x(p)}{px(p) - f - v(x(p))} \cdot \frac{\big(x(p) + (p - v'(x(p)))x'(p)\big)\,dp}{\big(x'(p)\big)\,dp} \\
\mathrm{OL}_2(p) &= \frac{1}{1 - \frac{f+v(x(p))}{px(p)}} \cdot \frac{1}{\frac{px'(p)}{x(p)}}. \qquad (2)
\end{aligned}$$

However, when they estimate operating leverage, Mandelker and Rhee use the elasticity of earnings with respect to changes in sales. The implied measure of operating leverage is

$$\begin{aligned}
\mathrm{OL}_3(p) &= \frac{\text{sales}}{\text{earnings}} \cdot \frac{d\,\text{earnings}}{d\,\text{sales}} \\
&= \frac{px(p)}{px(p) - f - v(x(p))} \cdot \frac{\big(x(p) + (p - v'(x(p)))x'(p)\big)\,dp}{\big(x(p) + px'(p)\big)\,dp} \\
\mathrm{OL}_3(p) &= \frac{1}{1 - \frac{f+v(x(p))}{px(p)}} \cdot \frac{1}{1 + \frac{px'(p)}{x(p)}}. \qquad (3)
\end{aligned}$$

The main difficulty with earnings-based measures is that they break down when earnings are negative or small and positive, both situations when the role of abandonment options becomes important.[4]

2.2 The expected rate of return

In the absence of an abandonment option, the firm generates a perpetual cash flow of $\pi(p_t)$. Standard methods can be used to show that the market value of the firm, $V(p)$, satisfies the ordinary differential equation[5]

$$0 = \frac{1}{2}\sigma^2 p^2 V''(p) + (\mu - \lambda)pV'(p) - rV(p) + \pi(p). \qquad (4)$$

The expected rate of return from owning such a firm equals

$$E[r_{\text{no flex}}] = \frac{1}{V(p)}\left(\pi(p) + \frac{E[dV(p)]}{dt}\right) = \frac{1}{V(p)}\left(\pi(p) + \mu pV'(p) + \frac{1}{2}\sigma^2 p^2 V''(p)\right) = r + \lambda \frac{pV'(p)}{V(p)},$$

where I have used (4) to complete the last step.

However, when the abandonment option is present, the owner of the firm will permanently abandon operations as soon as the output price falls below a threshold that is chosen to maximize the market value of the firm. I show in the appendix that if this threshold equals the constant $\hat{p}$, the market value of the firm equals

$$\begin{cases} S, & \text{if } p \le \hat{p}, \\ V(p) + (S - V(\hat{p}))\left(\dfrac{p}{\hat{p}}\right)^{-\beta}, & \text{if } p > \hat{p}, \end{cases} \qquad (5)$$

[4] However, as I show in Section 3, a non-monotonic relationship between the elasticity-based operating leverage measures and the expected rate of return is evident even when the firm's salvage value is sufficiently large.

[5] See, for example, Dixit and Pindyck (1994).

where

$$\beta = \frac{\mu - \lambda}{\sigma^2} - \frac{1}{2} + \sqrt{\frac{2r}{\sigma^2} + \left(\frac{\mu - \lambda}{\sigma^2} - \frac{1}{2}\right)^2} > 0. \tag{6}$$

The optimal abandonment threshold maximizes $(S - V(\hat{p}))\hat{p}^{\beta}$, and so is determined implicitly by the first order condition

$$S - V(p^*) = \frac{p^* V'(p^*)}{\beta}. \tag{7}$$

It follows that the market value of the firm, assuming the abandonment option is exercised optimally, equals

$$W(p) = \begin{cases} S, & \text{if } p \leq p^*, \\ V(p) + (S - V(p^*))\left(\frac{p}{p^*}\right)^{-\beta}, & \text{if } p > p^*. \end{cases}$$

Using the same approach as above, the firm's expected rate of return equals

$$E[r_{\text{flex}}] = r + \lambda \frac{pW'(p)}{W(p)}$$

when abandonment is allowed.

Note that the market value of the firm with the abandonment option equals

$$W(p) = V(p) + A(p),$$

where

$$A(p) = \begin{cases} S - V(p), & \text{if } p \leq p^*, \\ (S - V(p^*))\left(\frac{p}{p^*}\right)^{-\beta}, & \text{if } p > p^*, \end{cases}$$

is the market value of the firm's abandonment option. The expected rate of return for the abandonment option alone is

$$E[r_{\text{option}}] = r + \lambda \frac{pA'(p)}{A(p)} = r - \beta\lambda$$

prior to abandonment occurring (that is, when $p > p^*$). Because $\beta > 0$, the expected rate of return from holding just the abandonment option is less than the risk-free interest rate.

I show in the appendix that the expected rate of return for the firm with the abandonment option can be written as the following value-weighted average of the expected rate of return of the firm without the abandonment option and the expected rate of return of the abandonment option on its own:

$$E[r_{\text{flex}}] = \frac{V(p)}{V(p) + A(p)} E[r_{\text{no flex}}] + \frac{A(p)}{V(p) + A(p)} E[r_{\text{option}}]. \tag{8}$$

2.3 Comparative statics

Because all measures of operating leverage are endogenous, there can be no causal relationship between operating leverage and the expected rate of return. Therefore, I follow the approach of Booth (1991) and examine the effect of some exogenous change on both the degree of operating leverage and the expected rate of return. Let θ denote a parameter that describes some aspect of the firm's cost structure. Since changing θ has no impact on $E[r_{\text{option}}]$, its effect on the firm's expected rate of return is described by

$$\frac{\partial E[r_{\text{flex}}]}{\partial \theta} = \frac{V(p)}{V(p)+A(p)} \frac{\partial E[r_{\text{no flex}}]}{\partial \theta} + (E[r_{\text{no flex}}] - E[r_{\text{option}}]) \frac{\partial}{\partial \theta} \left(\frac{V(p)}{V(p)+A(p)} \right), \qquad (9)$$

as shown in the appendix. Thus, the expected rate of return's response to this change in the firm's cost structure has two components. First, the expected rate of return corresponding to the no-flexibility case changes. Second, the relative importance of the abandonment option will change—if the change in cost structure makes the option to abandon more valuable, the effect is to shift weight from the relatively-high cost of capital ($E[r_{\text{no flex}}]$) to the relatively low one ($E[r_{\text{option}}]$), reducing the firm's overall expected rate of return.

These two components of the change in the firm's expected rate of return are likely to work in opposite directions. This follows from the observation that the changes in the firm's cost structure that are associated with greater operating leverage will make the firm's cash flows more sensitive to exogenous shocks, and therefore more volatile. This greater volatility then makes the abandonment option more valuable.[6] Thus, the change in cost structure raises operating leverage as well as making the abandonment option more valuable: the first component in (9) is positive and the second is negative.

Consider the example of a change in the firm's fixed cost, f. Changes in f have no effect on optimal output $x(p)$. Therefore, they change all of the measures (1), (2), and (3) in the same direction, with higher values of f inducing greater operating leverage. However, the impact on the firm's expected rate of return is more complicated. The standard textbook explanation implies that the first component in (9) is positive—a larger fixed cost raises $E[r_{\text{no flex}}]$. However, a larger fixed cost will also make the option to abandon the firm more valuable, so the second component is negative. Although the sign of $\partial E[r_{\text{flex}}]/\partial f$ may be ambiguous for any particular value of f, it must be negative for at least some values of f. To see why, note that if the fixed cost is sufficiently low that the value of the abandonment option is close to zero, then $W(p)$ will be approximately equal to $V(p)$ and $E[r_{\text{flex}}]$ will approach $E[r_{\text{no flex}}]$, which will generally be greater than the risk-free interest rate. In contrast, if the fixed cost is sufficiently large that imminent abandonment is optimal then $W(p)$ will be approximately equal to S and quite insensitive to changes in the output price. In this case $E[r_{\text{flex}}]$ will be approximately equal to the risk-free interest rate. It follows that $E[r_{\text{flex}}]$ must be decreasing in f over some interval, for if this were not the case, $E[r_{\text{flex}}]$ could not possibly fall to r for large values of f. Over this interval, raising the fixed cost will simultaneously *raise* operating leverage and *lower* the expected rate of return.

I demonstrate the effect of changes in fixed costs, as well as other characteristics of the firm's cost function, using a specific example in the following section.

[6]The abandonment option is a put option that gives the firm's owner the right to sell the firm's future cash flows at a strike price equal to the firm's salvage value. Like all put options, its value is an increasing function of the volatility of the underlying asset (in this case, the right to the firm's future cash flows).

3 A particular example

In this section I consider the special case where the firm's variable cost function is $v(x) = wx^\alpha$, where $w > 0$ and $\alpha > 1$ are constants. The optimal production level is

$$x(p) = \left(\frac{p}{\alpha w}\right)^{1/(\alpha-1)}$$

and the firm's cash flow is

$$\pi(p) = (\alpha - 1)\left(\alpha^\alpha w\right)^{-1/(\alpha-1)} p^{\alpha/(\alpha-1)} - f$$

while it operates. It is easily shown that the market value of the firm without the abandonment option equals

$$V(p) = bp^{\alpha/(\alpha-1)} - \frac{f}{r},$$

where

$$b = \frac{(\alpha-1)^3\left(\alpha^\alpha w\right)^{-1/(\alpha-1)}}{(\alpha-1)^2 r - \alpha(\alpha-1)(\mu-\lambda) - \frac{1}{2}\alpha\sigma^2}.$$

It follows that the expected rate of return for the firm without the abandonment option is

$$E[r_{\text{no flex}}] = r + \lambda\frac{pV'(p)}{V(p)} = r + \left(\frac{\alpha\lambda}{\alpha-1}\right)\frac{1}{1 - \frac{f}{br}p^{-\alpha/(\alpha-1)}}. \tag{10}$$

Now consider the firm with the abandonment option. Substituting the solution for $V(p)$ into (7) and solving for p^* shows that the optimal abandonment threshold is

$$p^* = \left(\frac{\beta\left(S + \frac{f}{r}\right)}{b\left(\frac{\alpha}{\alpha-1} + \beta\right)}\right)^{\frac{\alpha-1}{\alpha}}.$$

After some tedious algebraic manipulation it follows that

$$E[r_{\text{flex}}] = \begin{cases} r, & \text{if } p \le p^*, \\ r + \left(\frac{\alpha\lambda}{\alpha-1}\right)\dfrac{1-\left(\frac{p^*}{p}\right)^{\frac{\alpha}{\alpha-1}+\beta}}{1-\frac{f}{br}p^{-\frac{\alpha}{\alpha-1}} + \frac{\alpha}{\alpha-1}\cdot\frac{1}{\beta}\left(\frac{p^*}{p}\right)^{\frac{\alpha}{\alpha-1}+\beta}}, & \text{if } p > p^*. \end{cases} \tag{11}$$

Comparing the expressions in (10) and (11) shows that recognition of the abandonment option reduces the firm's expected rate of return—the numerator in the risk premium of $E[r_{\text{flex}}]$ is smaller than in $E[r_{\text{no flex}}]$, while the denominator is larger, leading to an unambiguously lower expected rate of return. Equation (10) shows that $E[r_{\text{no flex}}]$ is an increasing function of f, as the conventional wisdom suggests. However, equation (11) shows that the effect of changes in the fixed cost on $E[r_{\text{flex}}]$ is more complicated. The numerator in the risk premium is (via p^*) clearly a decreasing function of f and, although some algebraic manipulation is required, it is straightforward to show that the denominator is also decreasing in f. The overall impact of an increase in f on $E[r_{\text{flex}}]$ is ambiguous.

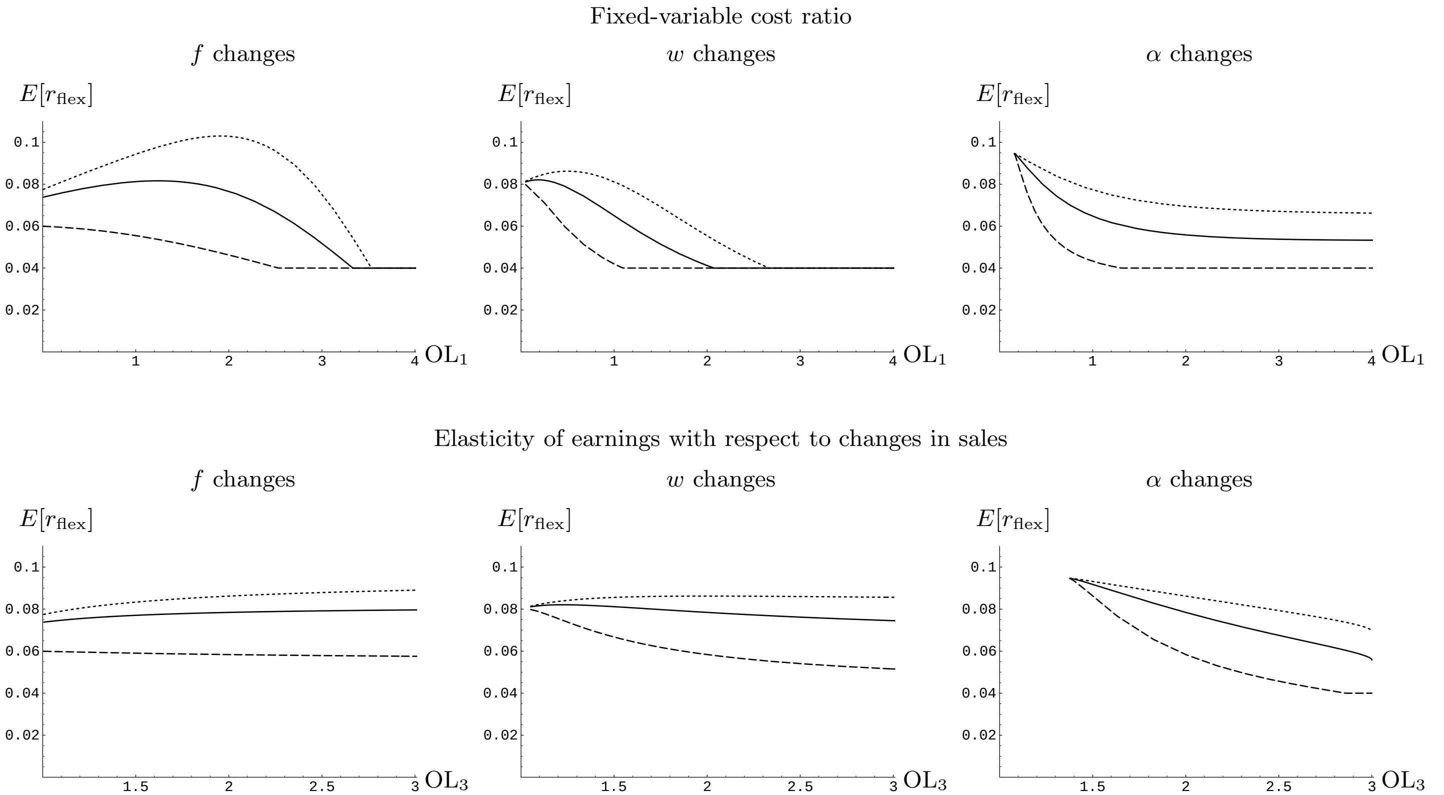
To further investigate the impact of changes in f on the expected rate of return I consider a numerical example in which $p = 1$, $\mu = 0$, $\sigma = 0.2$, $r = 0.04$, $\lambda = 0.02$, $f = 0.5$, $w = 0.25$, and $\alpha = 2$. Each graph in the top panel of Figure 1 plots the relationship between the expected rate of return (the vertical axis) and the fixed-variable cost ratio (the horizontal axis) as the indicated cost parameter is varied while holding all other parameters constant. The dotted curves correspond to a salvage value of $S = 5$, the solid curves to $S = 10$, and the dashed curves to $S = 30$.[7] Each graph in the bottom panel repeats this exercise, but using the elasticity of earnings with respect to changes in sales to measure operating leverage.

The two left hand graphs in Figure 1 show the relationship between the expected rate of return and the two operating leverage measures as the firm's fixed cost varies. Increasing the fixed cost has no impact on the firm's optimal output level, so variable costs are unaffected and the fixed-variable cost ratio increases. In the top graph the expected rate of return is an increasing function of the fixed cost for small values of f and a decreasing function for higher values. Increases in the fixed-variable cost ratio are associated with increases in the expected rate of return when the ratio is small, which, since it is in precisely these circumstances that the abandonment option is relatively unimportant, is not surprising. However, when the fixed-variable cost ratio is large, increases in the ratio are associated with reductions in the expected rate of return, confirming that the greater importance of the abandonment option can dominate the greater variability of short-term cash flows. The situation is not so clear in the bottom graph because, unless the salvage value is high, the expected rate of return only starts to fall once fixed costs have climbed past the point where earnings become negative (and the earnings elasticity is undefined). Raising the salvage value means that the turning point evident in the top graph occurs when earnings are still positive (and the elasticity still defined). This happens in the bottom curve in this graph, which shows a negative relationship between the elasticity-based operating leverage measure and the expected rate of return.

The two middle graphs in Figure 1 show the relationship between the expected rate of return and the two operating leverage measures as w varies. Unlike the case just described, this change affects the firm's output level, with $x(p)$ falling as w rises. However, variable cost $v(x(p))$ is decreasing in w, so that higher values of w are associated with a higher fixed-variable cost ratio. The top graph shows that here, too, the expected rate of return is positively related to operating leverage when the latter is low and negatively related to it when the latter is high. The bottom graph now shows the non-monotonic relationship between the expected rate of return and the elasticity-based measure of operating leverage. The two right hand graphs show the effect of raising α. Here, too, the parameter change affects the firm's output level, with $x(p)$ falling as α rises. In fact, higher values of α raise both measures of operating leverage. For the parameter values considered here, raising α leads to a lower expected rate of return in each case, so that both graphs show a negative relationship between operating leverage and the expected rate of return.

[7] Notice that in all cases as the salvage values increases (and the abandonment option becomes a more important component of the firm) the expected rate of return falls.

Figure 1: The relationship between operating leverage and the firm's expected rate of return



Notes. Each graph in the top panel plots the relationship between the expected rate of return (the vertical axis) and the fixed-variable cost ratio (the horizontal axis) as the indicated cost parameter is varied while holding all other parameters constant. The dotted curves correspond to a salvage value of $S = 5$, the solid curves to $S = 10$, and the dashed curves to $S = 30$. Each graph in the bottom panel repeats this exercise, but using the elasticity of earnings with respect to changes in sales to measure operating leverage. Other parameter values are $p = 1$, $\mu = 0$, $\sigma = 0.2$, $r = 0.04$, $\lambda = 0.02$, $f = 0.5$, $w = 0.25$, and $\alpha = 2$.

4 Concluding remarks

This paper demonstrates that increases in the operating leverage of a firm can be associated with reductions in the firm's expected rate of return, a result that contradicts the claim made in standard corporate finance textbooks that operating leverage and the expected rate of return should be positively related. All that is required to overturn the usual result is that the firm is able to cease operations if it becomes sufficiently unprofitable.

The expected rate of return of a firm with an abandonment option is a weighted average of (i) the expected rate of return of the firm without the abandonment option and (ii) the expected rate of return of the abandonment option (which is typically less than the risk-free interest rate). As fixed costs grow, the first component increases, as the textbooks argue, leading to a higher overall expected rate of return. However, the abandonment option also becomes more valuable, shifting more of the weight onto the latter component, and leading to a lower overall expected rate of return. In general, increases in fixed costs have an ambiguous impact on the expected rate of return of a firm. My analysis of a particular example shows that the second effect can dominate the first when operating leverage is high.

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Proofs

Proof of equation (5)

If $p \leq \hat{p}$, the firm ceases operations immediately, so that its market value, $\hat{V}$, must equal S. If $p > \hat{p}$, the firm continues to operate, receiving a cash flow of $\pi(p)\,dt$ over the next interval of length dt. Standard techniques show that $\hat{V}$ must satisfy the ordinary differential equation

$$0 = \frac{1}{2}\sigma^2 p^2 \hat{V}''(p) + (\mu - \lambda)p\hat{V}'(p) - r\hat{V}(p) + \pi(p)$$

in this region. Since $V(p)$ is a particular solution to this equation, the general solution is

$$\hat{V}(p) = V(p) + A_1 p^{-\beta} + A_2 p^{\gamma},$$

where β is given in (6) and

$$\gamma = -\left(\frac{\mu - \lambda}{\sigma^2} - \frac{1}{2}\right) + \sqrt{\frac{2r}{\sigma^2} + \left(\frac{\mu - \lambda}{\sigma^2} - \frac{1}{2}\right)^2} > 0.$$

When p is large, the market value of the abandonment option falls to zero, so that $\hat{V}(p) - V(p) \to 0$. This implies that $A_2 = 0$. Continuity of $\hat{V}(p)$ at $p = \hat{p}$ implies that $\hat{V}(p) \to S$ as $p \to \hat{p}+$, which in turn implies that $A_1 = (S - V(\hat{p}))\hat{p}^{\beta}$.

Proof of equation (8)

The expected rate of return for the firm with the abandonment option is

$$\begin{aligned}
E[r_{\text{flex}}] &= r + \lambda \frac{pW'(p)}{W(p)} \\
&= r + \lambda \frac{p(V'(p) + A'(p))}{V(p) + A(p)} \\
&= r + \lambda \frac{V(p)}{V(p) + A(p)} \frac{pV'(p)}{V(p)} + \lambda \frac{A(p)}{V(p) + A(p)} \frac{pA'(p)}{A(p)} \\
E[r_{\text{flex}}] &= \frac{V(p)}{V(p) + A(p)} E[r_{\text{no flex}}] + \frac{A(p)}{V(p) + A(p)} E[r_{\text{option}}].
\end{aligned}$$

Proof of equation (9)

Since $E[r_{\text{option}}]$ is independent of θ, it follows that

$$\begin{aligned}
\frac{\partial E[r_{\text{flex}}]}{\partial \theta} &= \frac{\partial}{\partial \theta} \left(\frac{V(p)}{V(p) + A(p)} E[r_{\text{no flex}}] + \frac{A(p)}{V(p) + A(p)} E[r_{\text{option}}] \right) \\
&= \frac{V(p)}{V(p) + A(p)} \frac{\partial E[r_{\text{no flex}}]}{\partial \theta} + E[r_{\text{no flex}}] \frac{\partial}{\partial \theta} \left(\frac{V(p)}{V(p) + A(p)} \right) \\
&\quad + E[r_{\text{option}}] \frac{\partial}{\partial \theta} \left(\frac{A(p)}{V(p) + A(p)} \right) \\
&= \frac{V(p)}{V(p) + A(p)} \frac{\partial E[r_{\text{no flex}}]}{\partial \theta} + E[r_{\text{no flex}}] \frac{\partial}{\partial \theta} \left(\frac{V(p)}{V(p) + A(p)} \right) \\
&\quad + E[r_{\text{option}}] \frac{\partial}{\partial \theta} \left(1 - \frac{V(p)}{V(p) + A(p)} \right) \\
\frac{\partial E[r_{\text{flex}}]}{\partial \theta} &= \frac{V(p)}{V(p) + A(p)} \frac{\partial E[r_{\text{no flex}}]}{\partial \theta} + \left(E[r_{\text{no flex}}] - E[r_{\text{option}}] \right) \frac{\partial}{\partial \theta} \left(\frac{V(p)}{V(p) + A(p)} \right).
\end{aligned}$$